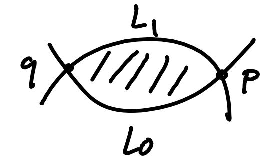
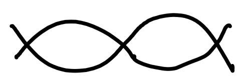
V. Gripp – 10/3/11 – obstruction in Floer theory

<u>Recall:</u> Lagr. Floer homology : $CF^{*}(L_0, L_1) = \Lambda^{|L_0 \cap L_1|}$



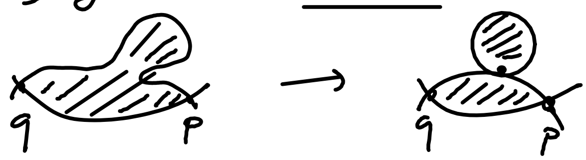
$$\partial p = \sum_{\substack{ind(u)=1 \\ [u]=\beta}} \#\mathcal{M}(p,q,J,\beta)/\mathbb{R} \; T^{\omega(u)} q$$

☆ To prove $\partial^2 = 0$, analyze ends of ind = 2 (1-dim!) moduli spaces.

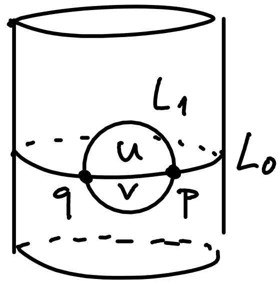
Ends might involve: → breaking of strips  → gives ∂^2 ✓

→ bubbling of spheres: expected codim. 2 ⇒ hopefully doesn't happen.

→ bubbling of discs : <u>real issue!</u>

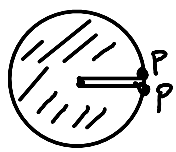


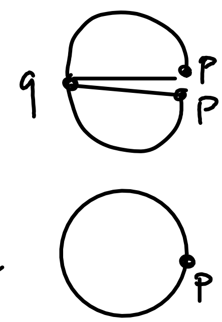
<u>Example:</u> $T^*S^1 = S^1 \times \mathbb{R}$:



$$\partial p = \pm T^{\omega(u)} q$$
$$\partial q = \pm T^{\omega(v)} p$$

So $\partial^2(p) \neq 0$!

Reason: $\mathcal{M}(p,p) = \left\{ \right.$  $\left. \right\}$ has ends



broken strip $\partial^2(p)$ ✓

constant strip + disc bubble

so contributions to ∂^2 don't cancel...

☆ What is $CF(L,L)$?

<u>Approach 1:</u> Hamiltonian perturbations : $CF(L,L) := CF(L, \phi_H(L))$

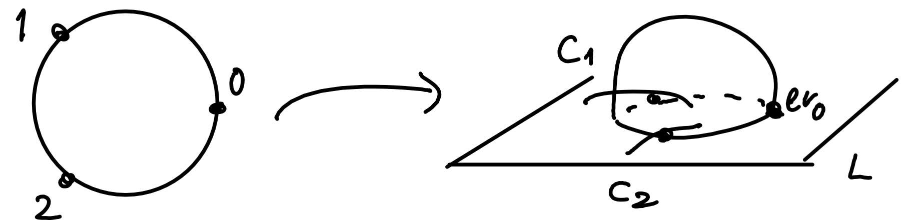
Difficulty: choose perturbations consistently wrt all compositions and higher operations.

[see eg. Seidel's book]

Approach 2: FOOO: $CF^{\alpha}(L,L) := C_*(L;\Lambda)$

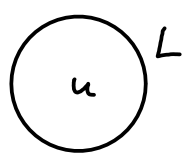
m_k := count holom. discs in (M,L) with incidence conditions on boundary marked pts.

e.g. $$m_2(C_2,C_1) := \sum_{\beta\in\pi_2(M,L)} ev_{0*}\Big([\bar{\mathcal{M}}_{0,3}(M,L;J,\beta)]\cap ev_1^{-1}(C_1)\cap ev_2^{-1}(C_2)\Big).T^{\omega(\beta)}$$

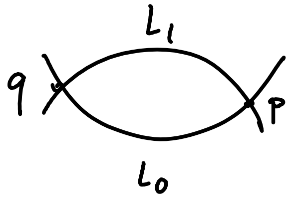


similarly m_k ... exception! $m_1(C) = \partial C + \sum_{\beta\neq 0}\Big(ev_{0*}[\bar{\mathcal{M}}_{0,2}]\cap ev_1^{-1}(C)\Big)T^{\omega(\beta)}$

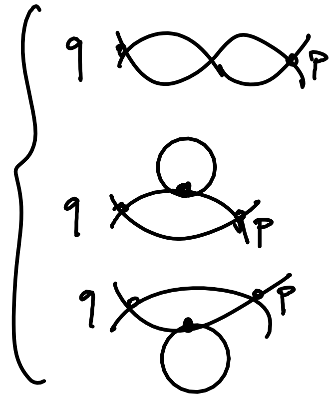
and $m_0 = \sum_{\beta\neq 0} ev_*[\bar{\mathcal{M}}_{0,1}(M,L;J,\beta)]\,T^{\omega(\beta)}$

Ex: $\mathbb{R}^2$,  $\Rightarrow m_0 = [L].T^{\omega(u)}$.

* going back to ∂^2:



boundary consists of



$\Rightarrow m_1(m_1(p)) \pm m_2(m_0^{L_1},p) \pm m_2(p,m_0^{L_0}) = 0.$